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OBSERVATION OF THE EARTH TIDAL EXTENSION AT WALFERDANGE

By

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Abstract

An observation of the earth tidal extension by means of extensometer which has been devised By I. Ozawa has been carried out in a gypsum mine at Walferdange, Luxemburg. The observation was begun in November, 1970. The sensitivity of the observation was in the range between 0.45×10^{-9}/mm and 0.61×10^{-9}/mm. The harmonic analyses have four periods whose lengths are from 30 days to 60 days. According to these analyses, the amplitude of M_2 wave and O_1 wave are $(0.72\text{–}1.84)\times10^{-8}$ and $(0.43\text{–}1.07)\times10^{-8}$, respectively. The sensitivity of the extensometer has been measured automatically with the carpaudin every other days.

The differences of the phase lags between M_2 wave's κ_{M_2} and O_1 wave's κ_{O_1} is given as $\kappa_{M_2}-\kappa_{O_1}=41.2°\sim43.4°$

The amplitude ratio O_1/M_2 of the O_1 wave's to M_2 wave's are given as $0.433\sim0.777$. From these result, we have the ratio l/h between Shida's number and Love's one is given as 0.114 ± 0.007. The probable values of these numbers is obtained as $h=0.75\pm0.06$ and $l=0.10\pm0.02$.

These results give the possibility that the values of Love's and the Shida's numbers are obtained directly by means of this observation far distant from the sea.

1. Introduction

I. Ozawa carried the extensometer which he devised from Japan to Belgium, and had a conference with P. Melchior to perform the tidal observation by means of the instrument at Walferdange, Luxembourg in 1969. The plan has carried out by Melchior et al., and the reading of the observation was sent to Ozawa by B. Ducarme. The final results should be summarized by the co-operations of these authors. Now the tentative result of four periods in the reading sent is summarized by Ozawa in this paper.

As the observatory at Walferdange is at more than 300 kilometers distant from the sea, the effect of loading caused by the oceanic tide seems to be very small. Then we neglect the effect of the load tide in the calculation of the direct effect of the astronomical tide. This plan is important and the first to realize an observation of the direct effect of the earth tidal extension.

It is important to study the effect of the load tide. To study the load tide is not

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our final object, but to research the direct effect of the earth tide. The best method for this study of the direct effect is to practise an observation at the center part of the continent. It is regretable that there is no observation at the center part of the continent. Perhaps the reason why there is no observatory is that the tidal strain has been thought by many scientists to be not so large as to observed with their observational technique. Nowadays, our observational technique can comply with our wishes easily. The obstacle caused form no commercially use less should be broken by our passions in natural science.

2. Observations

Walferdange is located in 6°09′17″ of east longitude, and 49°39′52″ of north latitude. This observatory is in the distance of more than 300 km from the sea. These seas are the North Sea and the English Channel. The observatory is in a gypsum mine. The gallery of the mine is divided into the working and un-working branches at about 50 m from the pit mouth. The observing room is in an un-working branch at a walk of 500 m from the mouth. There are few workers in this mine, and little digging. As the room is dry, we need not protect the instrument from the damp, and we need not worry about the damp over observational materials in a skin bag in the gallery.

The extensometer is put in the direction of N 38°58.7′ E–S 38°58.7′ W. The extensometer is an improved type of H-59-B type (I. Ozawa, [1960]) in order to obtain better calibration, namely, its size of the calibration system is greater than that of the former type. The standard scale of the instrument consists of a rod made from super-invar, which is 12 m long and 1 cm in diameter. The calibration system has a dial gauge of the screw type which is able to calibrate the relative displacement by super-invar, which is 12 m long and 1 cm in diameter. The calibration system has a dial gauge of the screw type which is able to calibrate the relative displacement by 0.7μ, and a plate of Verbaandert (crapaudine) (J. Verbaandert, [1959]) is added on the calibrating screw by Melchior. The bearing plate (crapaudine) is a disc which has a cave filled with mercury. The thickness of the crapaudine is calibrated by the pressure of the mercury in the cave. The variation of the thickness makes the small relative displacement for the calibration.

The relative sensitivity is usually obtained by means of measuring the period of the pendulum of the amplifier (I. Ozawa, [1970]). The curves of the tidal extensions are recorded on photographic paper. The instrument was set by Melchior et al., and the observation was started on November 4th, 1970. The observational operations have been mainly done by J. Flick. The reading of the records of 484.5 days in the period from February 7th, 1971 to September 15th, 1972 was sent to Ozawa by B. Ducarme.

Few accidents occurred in the curves of these records have some drifts in the sense of the extensions. The causes of the drift seems to be indicate that the suspension wires of the pendulum are thin, the weight of the pendulum is too heavy, the balance between the crapaudine and standard scale has creep, and the ditch where the in-

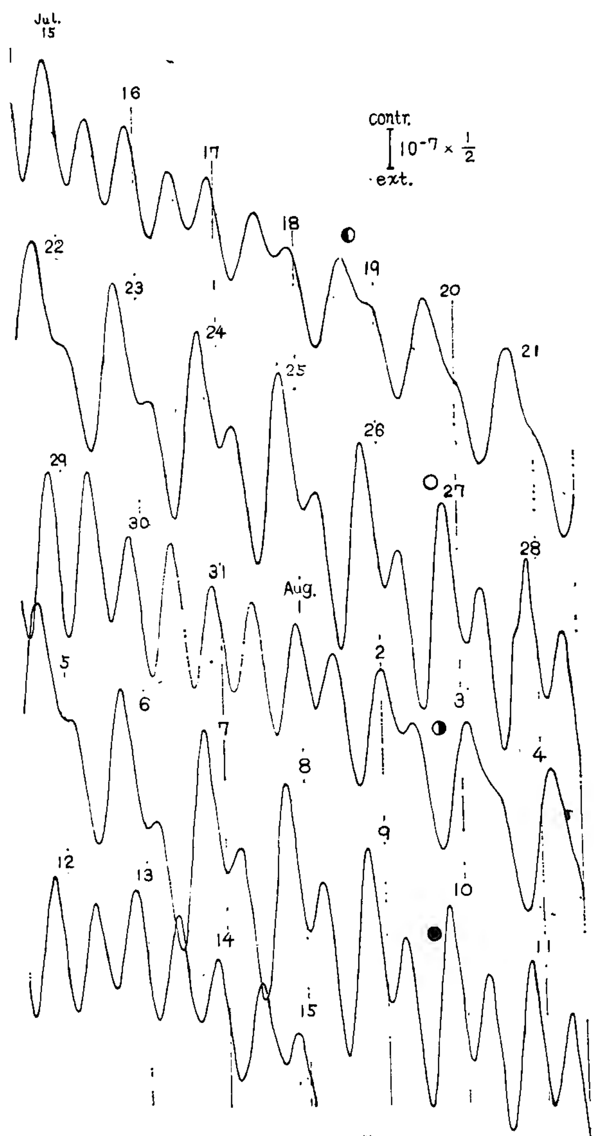

Fig. 1. Observed curves of the tidal extensions for the period of 30 days from December 25, 1972.

strument was set has creep. The drifts caused from the suspension wire and the weight of the pendulum can be easily corrected. But we have no obstacle from the dirft in the analyses of the earth tide because the drift is usually uniform. Fig. 1 shows the example of the observed curves of the extensometer for one month.

The sensitivity of the instrument has been measured automatically with the crapaudine every some days. These sensitivities are in the range from 0.45×10^{-9}/mm to 0.61×10^{-9}/mm.

The amplitude of the diurnal waves are much less than that of the semi-diurnal waves of the oceanic tide in the neighbouring sea. For example, the ratio of $(K_1+O_1)/(M_2+S_2)$ is less than 0.1 in most seas in this district. The amplitude of the semi-diurnal wave is not large in most of sea-regions except the some parts of the English Channel.

3. Analyses

We pick up four series of readings of observations which have no lack of the recordings. They are shown as follows,

for 30 days from June 3rd to July 2nd, 1971,

for 45 days from December 25th, 1971 to Feurbary 4th, 1972,

for 30 days from March 20th to April 18, 1972, and

for 60 days from July 15th to September 12, 1972.

The analyses of the short period (of 13 components tides) which are used at the Hydrographic Division of Maritime Safety Agency of Japan are done for 30 days' period, and the expanded method of Darwin's for those of the periods longer than one month is used. As the drift in these curves are linear and their gradients are almost constant, the elimination of the drifts not be carried out before superposing the readings in order to filter the special periods, but were to be done just before the

Table I. The analyzed values of the tidal extension and the amplitude ratio between the component tides, the ratio l/h calculated from the amplitude ratio O_1/M_2. And the diference between the phase lags of M_2 and O_1 and ratio l/h calculated from the diference, and the phase lags of M_2 and O_1.

Component tide et al.	For 45 days from Dec. 25, 1971.		For 30 days from Mar. 20, 1972.		For 30 days from Jun. 3, 1971.		For 60 days from Jul. 14, 1972.	
	Amplitude	Phase lags	Amplitude	Phase lag	Amplitude	Phase lag	Amplitude	Phase lag
M_2	$\times 10^{-8}$		$\times 10^{-8}$		$\times 10^{-8}$		$\times 10^{-8}$	
	1.84	6°3	0.72	7°3	0.73	2°3	1.15	6°9
S_2	0.49	250°1	0.32	8.3	0.34	14.6	0.61	41.6
K_2	0.13	250°1	0.09	8.3	0.09	14.6	0.17	41.6
K_1	1.14	328°0	0.54	304.0	0.91	341.2	0.69	299.7
O_1	1.07	325°1	0.56	321.3	0.43	319.6	0.73	323.6
P_1	0.38	328°0	0.18	304.0	0.34	341.2	0.23	299.7
O_1/M_2	0.583		0.777		0.592		0.633	
$l/h(O_1/M_2)$	0.124		0.060		0.116		0.103	
S_2/M_2	0.266		0.443		0.461		0.533	
S_2/K_1	0.430		0.584		0.370		0.892	
$\kappa_{M_2}-\kappa_{O_1}$		41°2		43°4		42°7		43°4
$l/h(\kappa_{M_2}-\kappa_{O_1})$		0.127		0.132		0.130		0.136
$l/h(M_2)$		0.073		0.081		0.072		0.085
$l/h(O_1)$		0.144		0.156		0.160		0.147

Fourier analyses supposing that the drifts are linear.

The arguments of the component tides are calculated by use of the Japanese Ephemeris in order to shut out large errors in the calculations.

The results of these analyses are shown in Table I.

4. Considerations

As Walferdange is at a distance of more than 300 km, the effect of the load tide is neglected in the following considerations.

The diurnal and semi-diurnal waves of the horizontal strain elements $e_{\theta\theta}$, $e_{\phi\phi}$ and $e_{\theta\phi}$ [I. Ozawa, 1959] are shown as

$$\begin{array}{lll} & \text{Diurnal-wave} & \text{Semi-diurnal-wave} \\ e_{\theta\theta} & (h-4l)A_1 \sin 2\theta \cos H, & (h \sin^2\theta + 2l \cos 2\theta) A_2 \cos 2H, \\ e_{\phi\phi} & (h-2l)A_1 \sin 2\theta \cos H, & [h \sin^2\theta - 2l(1+\sin^2\theta)] A_2 \cos 2H, \\ e_{\theta\phi} & 4l \sin\theta \sin 2\theta A_1 \sin H, & -4l \cos\theta \cdot \sin^2\theta A_2 \sin 2H. \end{array} \tag{1}$$

where h is Love's number, l is Shida's number, A_1 and A_2 are constants of the component tides, θ is the co-latitude, ϕ is the east longitude, H is the hour angle of the pending heavently body.

It finds that the coefficients of the terms of $\sin H$ and $\sin 2H$ don't contain the number h, but do number l only.

The extension $\varepsilon(\alpha)$ in the directions of an azimuth α is shown as

$$\varepsilon(\alpha) = e_{\theta\theta} \cos^2\alpha + e_{\phi\phi} \sin^2\alpha - e_{\theta\phi} \cos\alpha \sin\alpha \tag{2}$$

From (1) and (2), it finds also the coefficients of the sin-terms of the extension consist of only l of the earth tidal numbers in all the azimuths. If the accurate value of the sin-term is given, the value of l would be calculated theoretically. And the value of h is calculated also from the cos-term. But the volume of the sin-term is always much less than the cos-term in the observed values. And errors of the sin-terms depend not only on the observational amplitude, but also greatly on errors of the time scale of the observations and the analyses. At present, we can not depend much the values of l obtained from the sin-terms only. We must study other methods to obtain the reasonable values, h and l, and other volumes. The cos-terms consist of the linear combinations of h and l shown as formulae (1) and (2), and it can separate h and l by use of the cos-terms of the diurnal and the semi-diurnal waves.

Next, the amplitudes of the main component tides like M_2 and O_1 are able to be analysed accurately, and so, we can calculate the accurate value of the ratio l/h by use of the amplitude ratio O_1/M_2. The curve of O_1/M_2 versus to l/h at any latitude is given as parabola, and that at latitude of 49°39′52″ is shown in Fig. 2. The solid points on the curve are analysed values, and three points among fours' are within the range from 0.103 to 0.124.

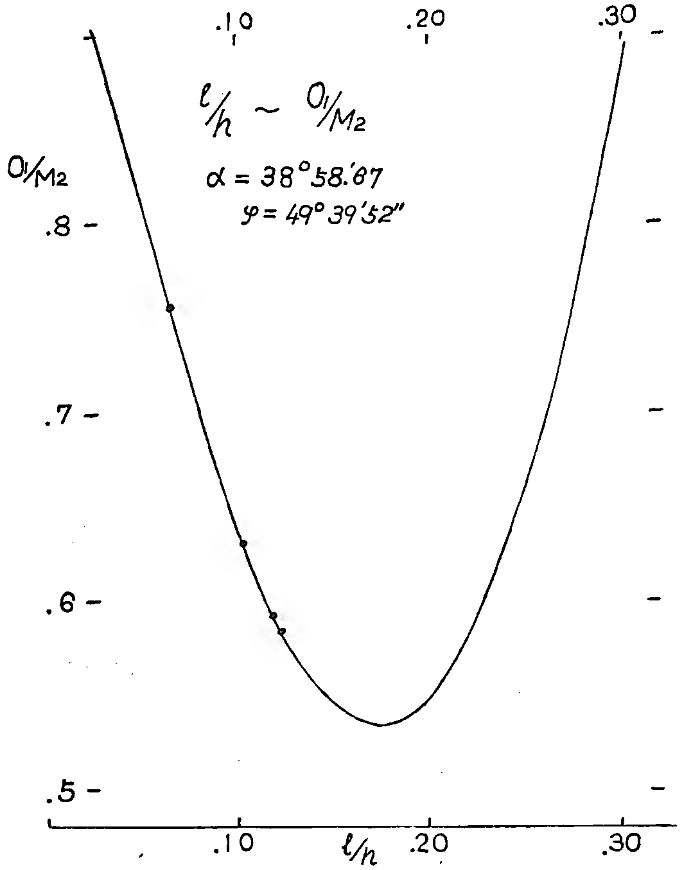

Fig. 2. The theoretical curve of the amplitude ratio O_1/M_2 versus the ratio l/h in the azimuth of 38°58.67′, and in the latitude of 49°39′52″.

The curves of the theoretical relations the phase lags of M_2 and O_1 waves versus l/h at the latitude of 49°39′52″ are shown in Fig. 3. The analysed values of the phase lags themselves may have some error, and are not always accurate. Then, the curves of the difference $\kappa_{M_2}-\kappa_{O_2}$, between the phase lags of M_2 and O_1 versus the ratio l/h is shown also in Fig. 3. According to Fig. 3, the ratio l/h except one seems to be in the range from 0.068 to 0.077 in the phase lags of M_2, to be in the range from 0.144 to 0.160 in those of O_1, and to be in that from 0.127 to 0.136 in those of $\kappa_{M_2}-\kappa_{O_2}$, respectively.

We can also calculate these numbers from the formulae of the cos and sin-terms of the diurnal and semi-diurnal waves, for example M_2 and O_1. We have $h=0.75\pm0.06$, and $l=0.10\pm0.02$ by means of this method.

The analyser estimates that the most probable value is ratio l/h obtained by means of the amplitude ratio O_1/M_2, and the mean value of the three which are near each other is 0.114 ± 0.007.

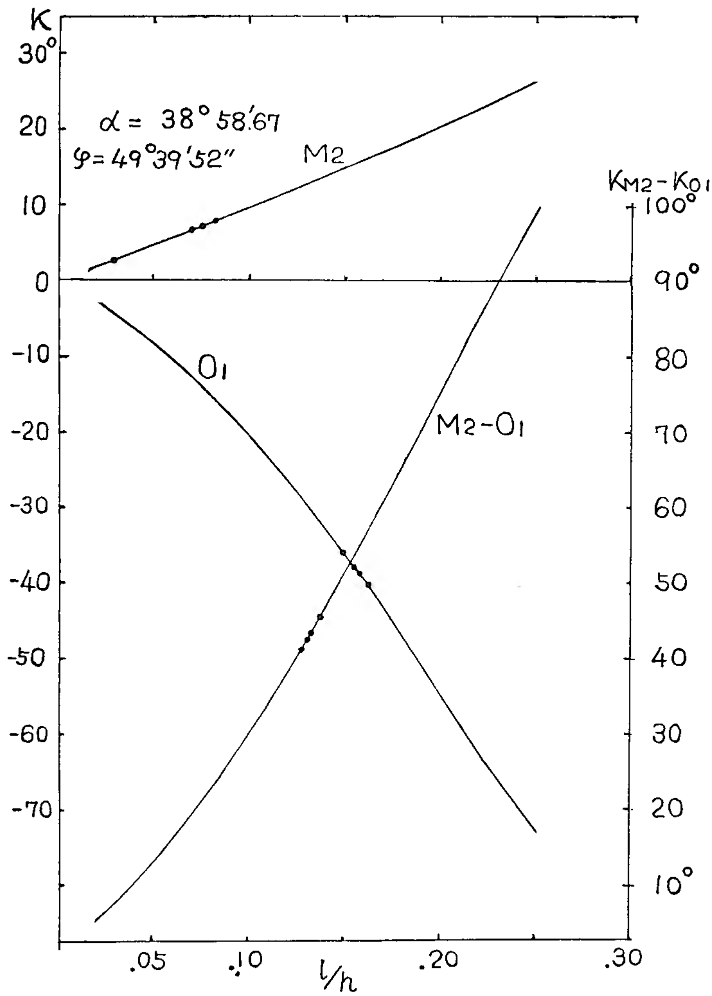

Fig. 3. The theoretical curves of the phase lags of M_2 and O_1, and the difference $\kappa_{M_2}-\kappa_{O_1}$ between the phase lags of M_2 and O_1 in the azimuth of 38°58.67′, and in the latitude of 49°39′52″.

Finally, the author wishes to have the observations in various directions, and to have more reasonable values.

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